Cabello's Nonlocality and Linear Optics

In a recent followup [1] to his elegant demonstration of quantum nonlocality without inequalities and without probabilities for two observers [2], Cabello has derived an experimentally testable Bell-type inequality that is violated by quantum mechanics. He has also proposed a way of testing this inequality purely by means of linear optics. This Comment challenges this proposal and argues that nonlinear optics is in fact required for a loophole-free demonstration of Cabello's nonlocality.

In Cabello's scheme, four qubit particles, labeled 1,2,3,4, in the state $|\psi^-\rangle_{13} \otimes |\psi^-\rangle_{24}$ (where $|\psi^-\rangle_{ij}$ is the singlet Bell state) are shared by two distant observers: Alice, who has qubits 1 and 2, and Bob who has qubits 3 and 4. Alice and Bob perform a number of measurements on their sets of qubits, whose results, when put together, lead to the conclusion of nonlocality.

My argument concerns one of these measurements, namely the one described by Eq. (10) of the paper [1]. While Alice measures the product of observables $\hat{z}_1\hat{z}_2$ and $\hat{x}_1\hat{x}_2$ on her pair of particles 1 and 2, Bob measures the product of $\hat{z}_3\hat{x}_4$ and $\hat{x}_3\hat{z}_4$ on his particles 3 and 4. For the state $|\psi^-\rangle_{13} \otimes |\psi^-\rangle_{24}$ quantum mechanics predicts anticorrelation between the two measurement results.

For the purpose of this Comment I will concentrate on Alice's apparatus. The operators $\hat{A} = \hat{z}_1\hat{z}_2$ and $\hat{B} = \hat{x}_1\hat{x}_2$ commute and can in principle be measured simultaneously on the same pair of particles. Such a measurement is, however, equivalent to making a full distinction between the four Bell states on both pairs of particles, and requires nonlinear optical interaction at a single-photon level as discussed extensively in the literature.

Cabello proposes to deal with this issue by noticing that instead of performing two simultaneous quantum measurements of *both* observables $\hat{A}$ and $\hat{B}$, their product $\hat{C} = \hat{A} \cdot \hat{B}$ can be determined as a *single* observable in a single quantum measurement. This latter observable is dichotomic and does not require nonlinear optics for its determination.

It is this proposed simplification that raises my concern. The measurement of $\hat{C}$ Alice performs when checking Eq. (10) is done using a different experimental arrangement as compared to the measurements of $\hat{A}$ and $\hat{B}$ done when checking Eqs. (6) and (7), respectively. How do we convince someone who does not believe in quantum mechanics that the measured observable $\hat{C}$ is always equal to the product of $\hat{A}$ and $\hat{B}$?

To visualize this argument, one can think of Alice's apparatus as a "black box" containing seven buttons: (1) measure $\hat{x}_1$; (2) measure $\hat{x}_2$; (3) measure $\hat{z}_1$; (4) measure $\hat{z}_2$; (5) measure $\hat{A}$; (6) measure $\hat{B}$; (7) measure $\hat{C}$.

In each of the measurements designated by Eqs. (2)–(10), she pushes the relevant button and reads out the measurement result. She does not know, however, what is inside the box, and cannot verify that the quantity she measures using button 7 is indeed the product of the quantities she measures using buttons 5 and 6. The logical sequence used in the paper [1], on the other hand, seems to require this piece of information to demonstrate nonlocality.

Note that if we do use nonlinear optics, this problem disappears. In this case the apparatus has only the first 6 buttons, and buttons 5 and 6 can be pushed simultaneously (yielding 2 values at the same time) when checking Eq. (10). A separate, 7th button, intended for measuring $\hat{A} \cdot \hat{B}$, is not necessary.

In the reported experiments towards complete discrimination of the four Bell states only one of the two "particles" was a single-photon state; the other one was a relatively strong coherent pulse [3]. To my knowledge, such experiments with single photons as both Bell's particles have not been performed. Evaluating the validity of the linear-optical version of Cabello's gedanken experiment is thus of high importance as it establishes the general possibility of implementing it with existing technological tools. In my opinion, while Cabello's nonlocality is undoubtedly a new and interesting observation, it cannot be confirmed experimentally at the present level of quantum optical technology.

A. I. Lvovsky*
Fachbereich Physik
Universität Konstanz
D-78457 Konstanz, Germany

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*Email address: Alex.Lvovsky@uni-konstanz.de

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