New results in optical homodyne tomography and their applications to quantum information technology

A. I. Lvovsky, S. A. Babichev, J. Appel,
Department of Physics and Astronomy, University of Calgary, Calgary AB T2N 1N4, Canada
lvov@ucalgary.ca

We present experimental homodyne tomography of an optical quantum bit represented by a single photon split into two optical modes. Fundamental and applied implications of the experiment are discussed.

In the optical implementation of the qubit, the logical value is assigned to a single photon being in one of two orthogonal modes *A* or *B*:

$$\left|\tilde{1}\right\rangle = \left|1_A, 0_B\right\rangle, \quad \left|\tilde{0}\right\rangle = \left|0_A, 1_B\right\rangle \tag{1}$$

where the right-hand side is written in the photon number (Fock) basis for each mode. Such a dual-rail optical qubit is an intuitively simple object, yet it is highly immune to decoherence and permits construction of efficient computational gates involving only linear optical elements.

Efficient application of optical qubits for quantum information processing is impossible without a robust technology of their *characterization*. To date, characterization of optical logical ensembles has been based on studying relative photon number statistics in each mode and in their various linear superpositions as well as (in the case of multiple qubits) photon number correlations between modes. This approach is based on postselection and does not provide a correct performance estimate of an experimental scheme under investigation, in particular, it does not allow one to evaluate its scalability.

Optical homodyne tomography, the quantum state characterization method based on studying quantum statistics of the electric field amplitude associated with the ensemble in question, is free from this disadvantage. Here we apply this technique to an optical qubit and evaluate its density matrix [1].

A two-mode qubit is generated when a single-photon state $|1\rangle$ incident upon a beam splitter entangles itself with the vacuum state $|0\rangle$ entering the other beam splitter input. Our goal was to characterize the two-mode state at the output, theoretically given by

$$|\Psi\rangle = \tau\left|1_A, 0_B\right\rangle - \rho\left|0_A, 1_B\right\rangle, \tag{2}$$

τ^2 and ρ^2 being, respectively, the beam splitter transmission and reflectivity. To perform tomography measurements, we have placed two homodyne detectors (associated with fictitious observers Alice and Bob) into each beam splitter output channel [Fig. 1(a)]. With every incoming photon, both detectors made a measurement of the field quadrature X_A and X_B with the local oscillators' phases set to θ_A and θ_B, respectively. The quadrature statistics collected at various phases were used to reconstruct the density matrix of the two-mode state.

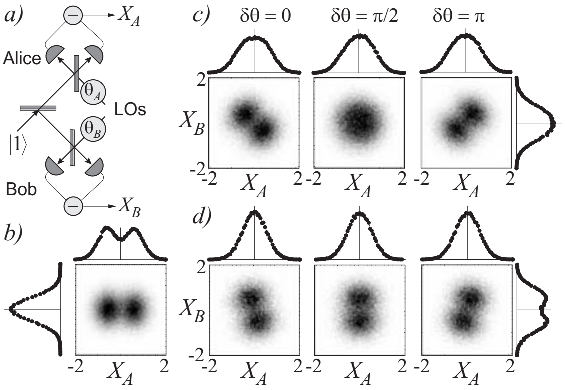

Figure 1. (a) Scheme of the experimental setup. LOs: local oscillators. (b-d) Histograms of the experimental quadrature statistics pr (X_A, X_B) for the "zero reflectivity" (b), symmetric (c), and highly reflective (92%, d) beam splitters.

In our experiment, the initial single-photon state was prepared by means of a conditional measurement on a biphoton produced via parametric down-conversion. We used frequency-doubled 2-ps pulses from a mode-locked Ti:Sapphire laser running at λ = 790 nm which underwent down-conversion in a BBO crystal, in a type-I frequency-degenerate configuration. The same laser was used as the local oscillator source for homodyne measurements. More details on our experimental setup can be found in Ref. [2].

We executed two data acquisition runs using two different beam splitters with transmissions τ^2 equal to 0.5 and 0.08. We varied the relative phase

$\delta\theta = \theta_A - \theta_B$ of Alice's and Bob's local oscillators slowly over a 2π range, and acquired about 300,000 pairs (X_A, X_B) from the homodyne detectors. Because the qubit was generated by splitting a single photon which has no optical phase, the sum of phases $\theta_A + \theta_B$ is meaningless and does not affect the homodyne statistics. Therefore, we let this phase vary randomly during the run.

The reconstructed density matrix [Fig. 2] features a strong contribution of the double-vacuum term $|0,0\rangle\langle 0,0|$. This is a consequence of imperfect preparation of the initial single photon: instead of the state $|1\rangle$, a statistical mixture

$$\hat{\rho}_{|1\rangle} = \eta|1\rangle\langle 1| + (1-\eta)|0\rangle\langle 0| \tag{3}$$

is available at the beam splitter input [2]. The vacuum fraction is directly transferred to the dual-mode ensemble:

$$\hat{\rho}_{|1\rangle} = \eta|\Psi\rangle\langle\Psi| + (1-\eta)|0,0\rangle\langle 0,0|. \tag{4}$$

The reconstructed ensemble is in excellent agreement with the above equation and corresponds to the preparation efficiency $\eta = 0.64$. The presence of the vacuum term demonstrates the method's ability to achieve *full* reconstruction of the qubit (including detection of undesired terms).

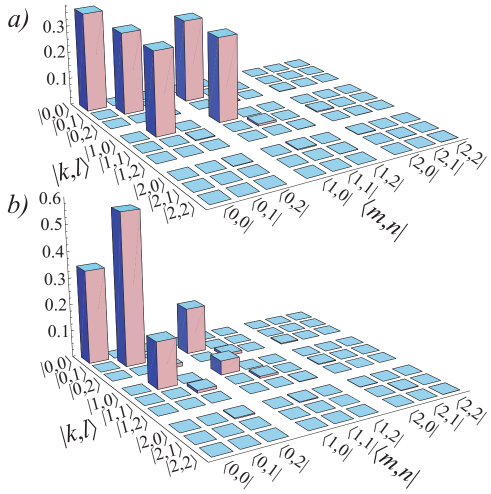

Figure 2. Density matrix (absolute values) of the measured ensemble for the symmetric (a) and highly reflective (b) beam splitters in the photon number representation.

The state (1) is entangled, i.e. it should exhibit noncontextually nonlocal properties when measured in a certain basis. And indeed, as we demonstrate in Ref. [1], by filtering quadrature measurements using a threshold discriminator and assigning them binary values according to their sign, one can demonstrate a violation of the Bell inequality, albeit with a loophole similar to the detection loophole in photon counting experiments.

Our experimental results can also be interpreted as an implementation of quantum communication protocol known as remote state preparation (RSP) [3]. In our present experimental arrangement, RSP is implemented as follows. With each incoming photon, Alice performs a homodyne measurement on her part of the entangled state (2) with the local oscillator set to a preselected phase θ. If her measurement result approximates some preselected value Q, she notifies Bob via a classical channel. Upon receipt of Alice's message, Bob performs a homodyne measurement of his fraction of $|\Psi\rangle$ to characterize the remotely prepared state.

By detecting a particular quadrature value $X_A = Q$ at the local oscillator phase θ, Alice projects the entangled resource (1) onto a quadrature eigenstate $\langle Q|_A$:

$$|\psi_B\rangle = {}_A\langle Q|\Psi\rangle = \tau\langle Q|1\rangle_A|0\rangle_B - \rho\langle Q|0\rangle_A|1\rangle_B, \tag{5}$$

which is just a coherent superposition of the single-photon and vacuum states

$$|\psi_B\rangle = x|0\rangle + y|1\rangle \tag{6}$$

with $x = \tau\langle Q|1\rangle_A$ and $y = -\rho\langle Q|0\rangle_A$.

In summary, we have characterized a two-mode optical qubit using homodyne tomography. For the first time, complete information about this quantum ensemble is revealed, including those terms that do not belong to the qubit subspace of the Hilbert space. The experimental data demonstrate a nonlocal character of the delocalized single photon in the form of the Bell inequality violation. Our experiment can be interpreted as postselected remote state reparation of single-mode photonic quantum bits in a counterintuitive scheme. Our work demonstrates the potential of combining discrete and continuous variable techniques in quantum information technology applications.

1) S. A. Babichev, J. Appel, A. I. Lvovsky, Phys. Rev. Lett. **92**, 193601 (2004)

1) A. I. Lvovsky et al., Phys. Rev. Lett. **87**, 050402 (2001); A. I. Lvovsky, S. A. Babichev, Phys. Rev. A **66**, 011801 (2002);T. Aichele, A. I. Lvovsky, and S. Schiller, Eur. Phys. J. D 18, 237 (2002).

3) S. A. Babichev, B. Brezger, A. I. Lvovsky, Phys. Rev. Lett. **92**, 047903 (2004)